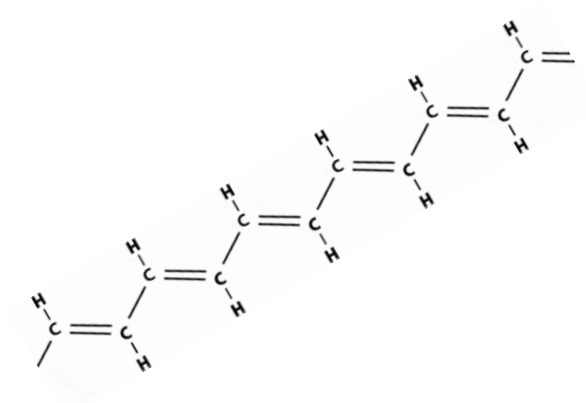
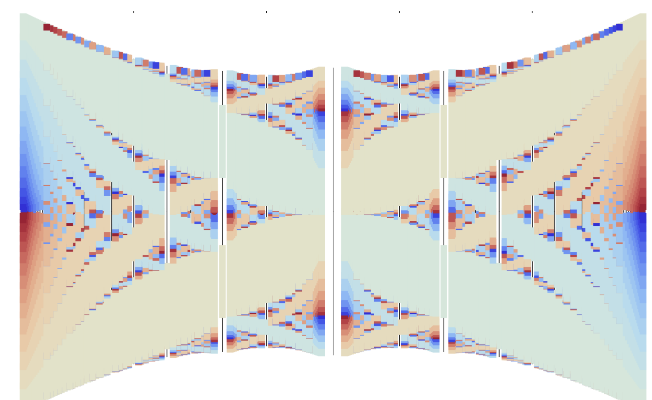
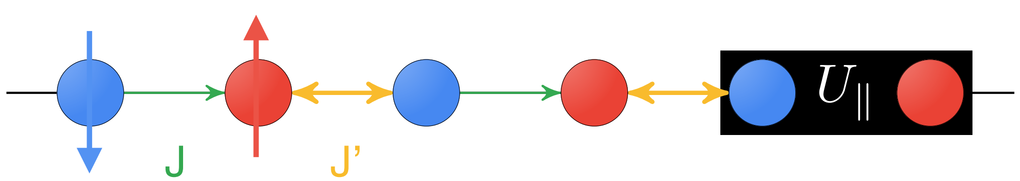
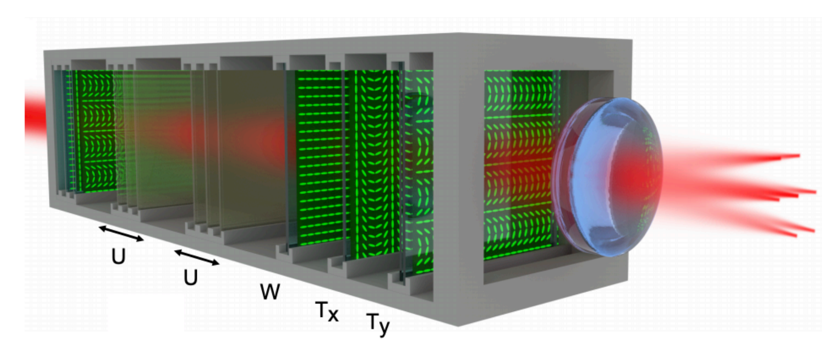
Detecting topological features in real space and time

Pietro Massignan









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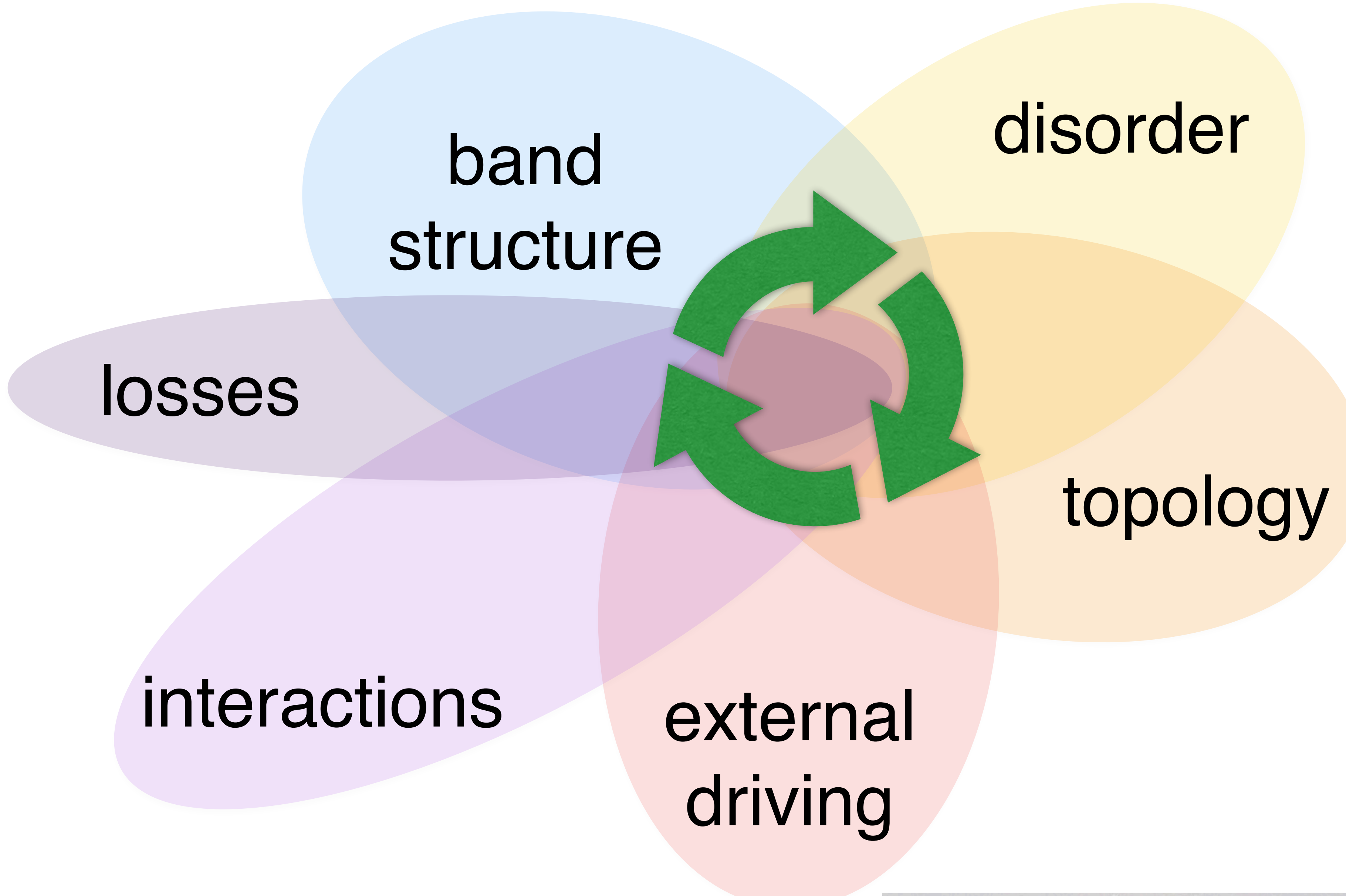
BARCELONATECH

ICFO
The Institute of Photonic Sciences

CIRM (Marseille) - 21/01/2020

Condensed matter



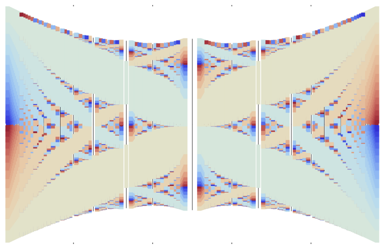
Plenty of emergent phenomena!

But we need to *quantify* these…

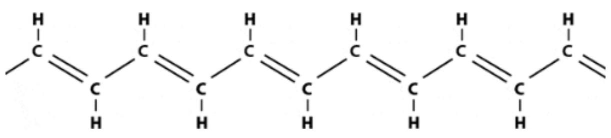
E.g., how to “detect topology”?

Outline

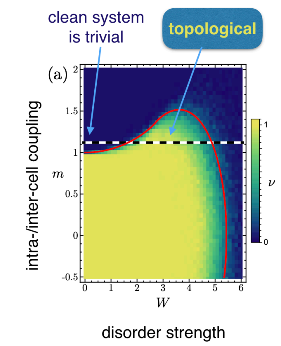
- Topology in condensed matter



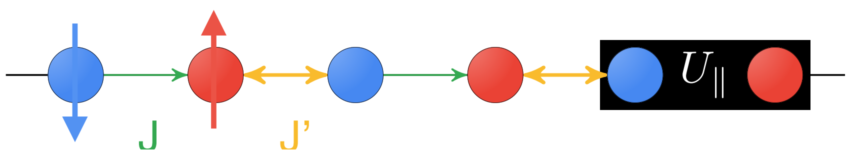
- One-dimensional chiral models



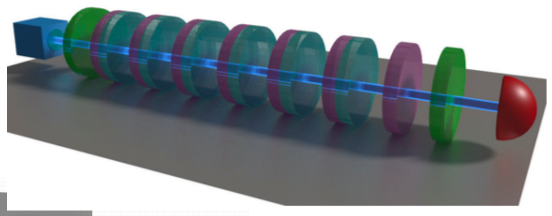
- Quantum Simulation:
  - ✦ Topological Anderson Insulator



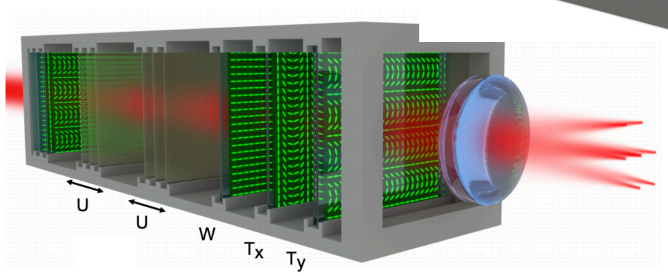
  - ✦ Interacting fermionic chains



- Quantum Walks ⟷ periodically-driven systems:
  - ✦ Chiral insulators in 1D QWs



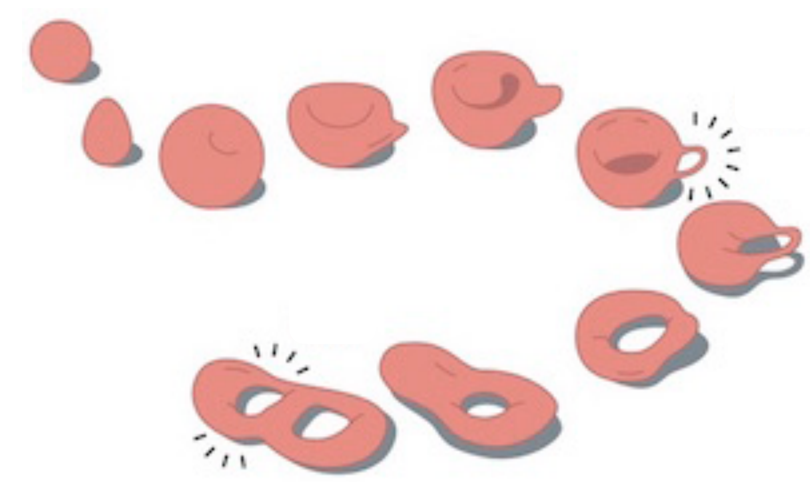
  - ✦ Chern insulators in 2D QWs



Topology

- Classification of objects and manifolds under continuous deformations

  ✓ stretch and bend
  ✘ but don't cut, puncture, or glue

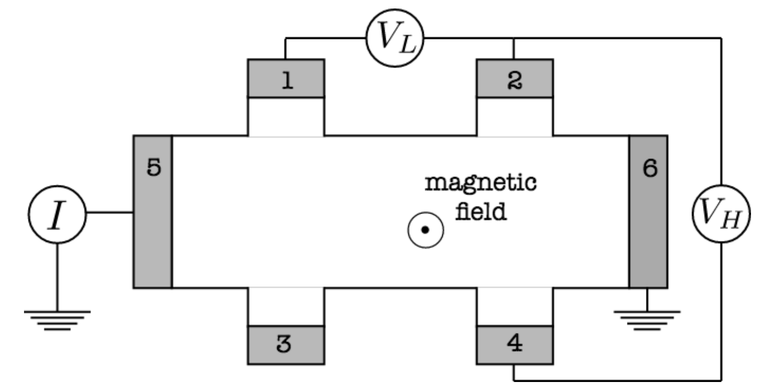


- *Global* properties!

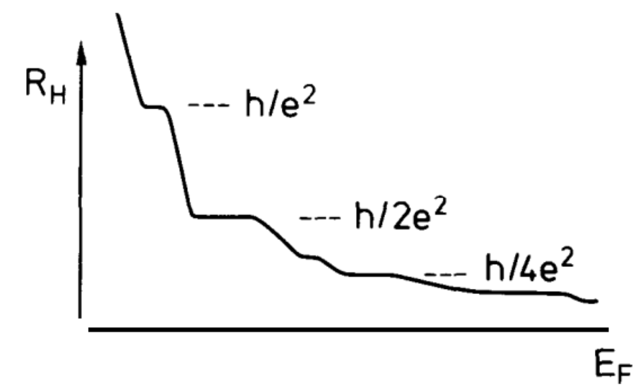
- Genus (# of holes)

- Winding number of a closed path
  (# of times it encircles a given object)

Hall effect



- Classical Hall effect (1879):
when current flows in a 2D material,
in presence of an out-of-plane B field,
there appears a transverse (Hall) current



K. Von Klitzing
Nobel lecture

- Quantum Hall effect (1980):
at low temperatures and high-B,
the Hall current is quantized!

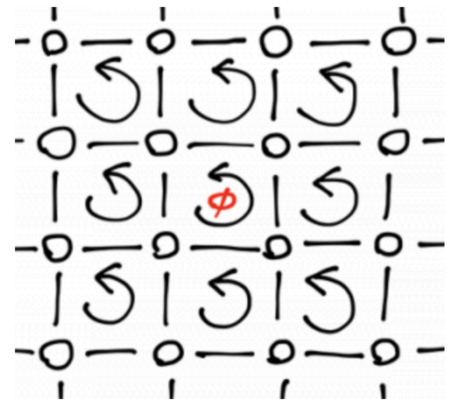
- Laughlin (1982): robustness due to **topology**

- TKNN (1982): Kubo formula links conductivity to *Chern numbers*
(topological invariants defined on the occupied bands).

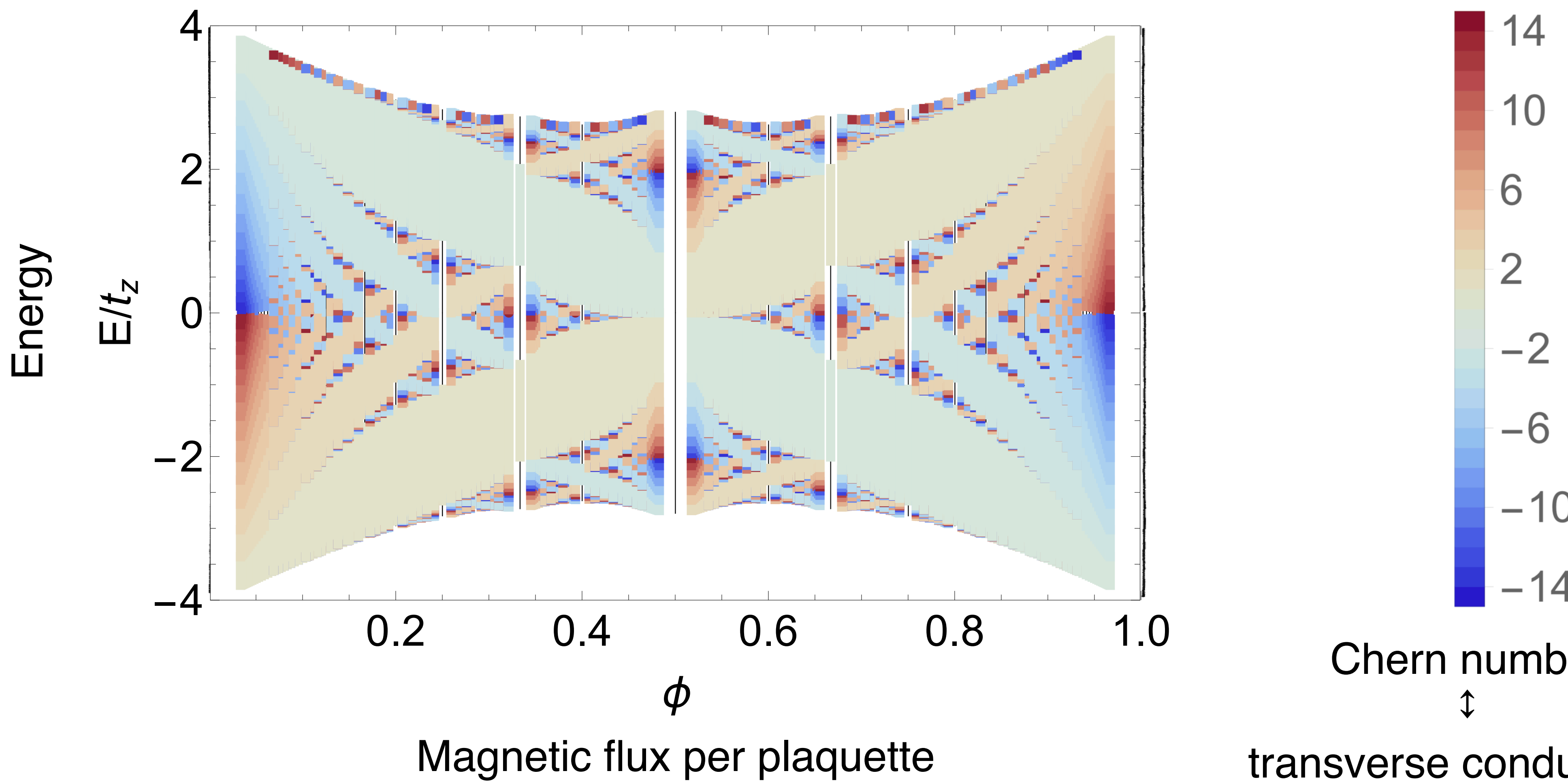
Thouless, Kohmoto, Nightingale & den Nijs
Phys. Rev. Lett. (1982)

Hofstadter butterfly

A very simple problem
hosting a *fractal* spectrum
with *topological* meaning



D. Hofstadter,
Phys. Rev. B (1976)



Chern number ν
↕
transverse conductivity $\nu e^2/h$

Topological insulators

- Insulators in the bulk, with few conducting modes on their edges.
- Protected by the band topology vs. local perturbations, like *disorder* and *defects.*
- *Bulk/edge correspondence:* edge modes intimately related to topological invariants.
- Enormous progresses in the last 20 years (QSH, 3D TIs., 4D QH, …)
- Characterization of non-interacting TIs in terms of discrete symmetries
  T: time-reversal
  C: charge-conjugation
  S: chiral

IQHE, Hofstadter, Chern insulators → A

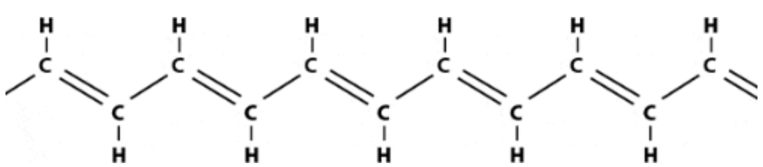
chiral → AIII, BDI

Class	T	C	S	# of dimensions 0	1	2	3	4	5	6	7
A	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}$	Chern number			$\mathbb{Z}$	0
AIII	0	0	1	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$
AI	$+$	0	0	$\mathbb{Z}$	0	Winding		$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
BDI	$+$	$+$	1	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$
D	0	$+$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0
DIII	$-$	$+$	1	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$
AII	$-$	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0
CII	$-$	$-$	1	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0
C	0	$-$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0
CI	$+$	$-$	1	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$

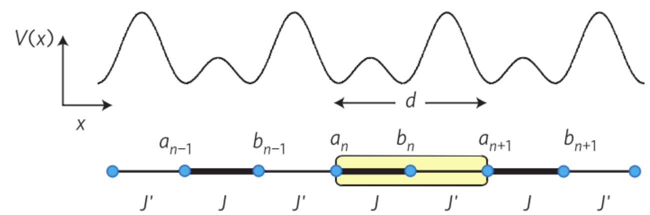
- Beyond the periodic table:
  Mott / crystalline / Anderson / Floquet TIs, …

Chiu, Teo, Schnyder & Ryu, Rev. Mod. Phys. (2016)

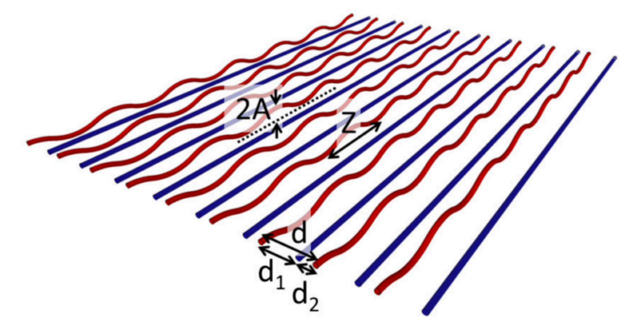
1D chiral systems



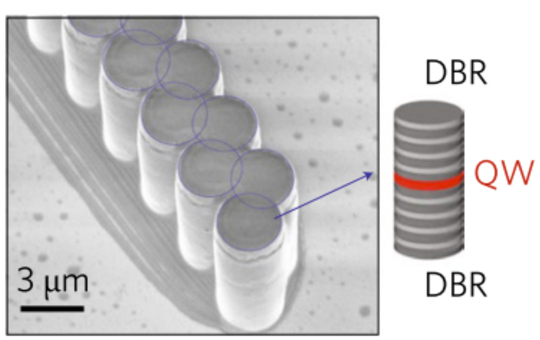
polyacetilene
[Nobel prize in Chemistry 2000]



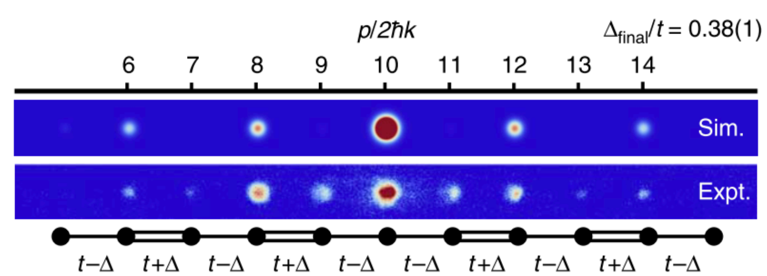
ultracold atoms
in superlattices
[M. Atala *et al.*, Nature Phys. 2013]



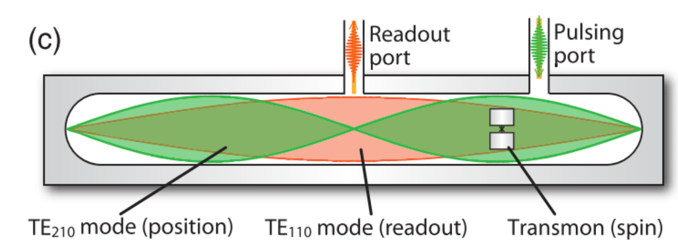
optical waveguides
[Zeuner *et al.*, PRL 2015]



cavity polaritons
[St. Jean *et al.*, Nature Phot. 2017]



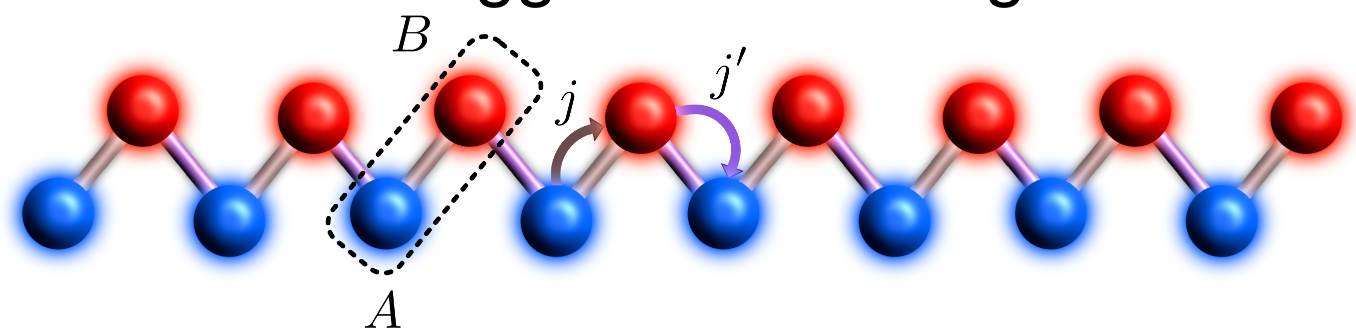
ultracold atoms
in momentum-lattices
[Meier *et al.*, Nature Comm. 2016]



SC qubits
in mw-cavities
[Flurin *et al.*, PRX 2017]

SSH model

- Spinless fermions with staggered tunnelings:



Su, Schrieffer & Heeger
Phys. Rev. Lett. (1979)

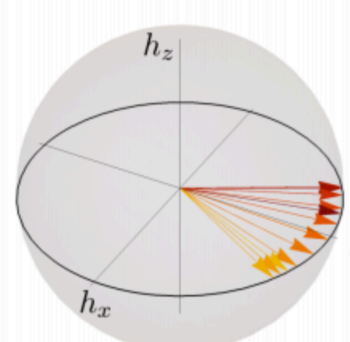
Asbóth, Oroszlány, & Pályi
Lecture Notes in Physics (2016)

- ∃ two sublattices
  ∃ a "canonical basis" where H is purely off-diag: $H = \begin{pmatrix} 0 & h^\dagger \\ h & 0 \end{pmatrix}$

- Chiral symmetry: $\Gamma H \Gamma = -H$ (Γ: unitary, Hermitian, local) $\Gamma = \sigma_z$

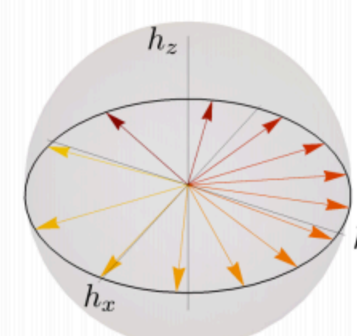
- In momentum space: $H_k = E_k \mathbf{n}_k \cdot \boldsymbol{\sigma}$ with $\mathbf{n}_k \perp \hat{\mathbf{z}}$ $\forall k$

- Winding:



$j' < j$

$\mathcal{W} = 0$



$j' > j$

$\mathcal{W} = 1$

- Bulk-edge correspondence: open-ended chains have $2\mathcal{W}$ edge modes

The winding *W*

- $\mathcal{W}$ may be calculated:

$$H_k = E_k \mathbf{n}_k \cdot \boldsymbol{\sigma}$$

  - from n : $\mathcal{W} = \oint \frac{\mathrm{d}k}{2\pi} (\mathbf{n} \times \partial_k \mathbf{n})_z = \oint \frac{\mathrm{d}k}{2\pi} \partial_k \phi$
  - from the *eigenstates* : $\mathcal{W} = \oint \frac{\mathrm{d}k}{\pi} \mathcal{S},$ $\quad \mathcal{S} = i \langle \psi_+ | \partial_k \psi_- \rangle$

skew polarization

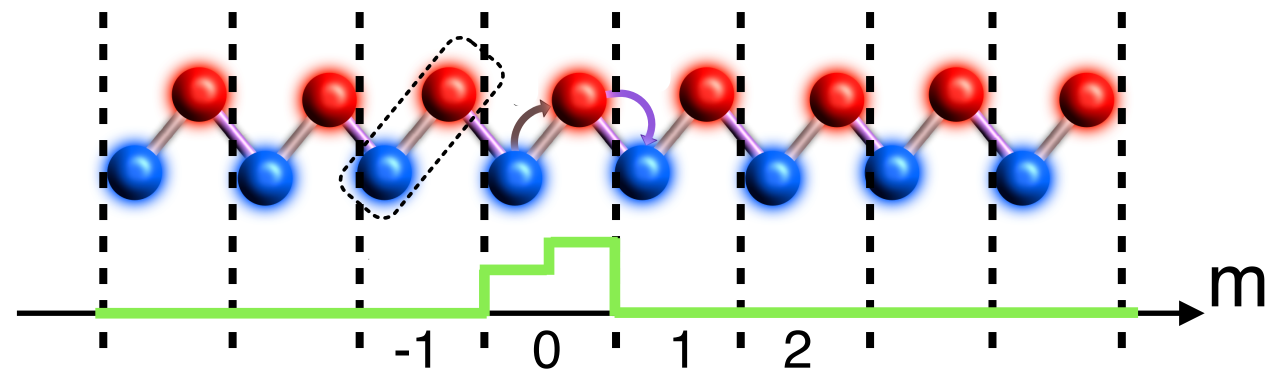
- What if the Hamiltonian is not known?
  Can one *measure* the winding?

  Yes, and it's simple!

Evolution in real time

- Initial condition

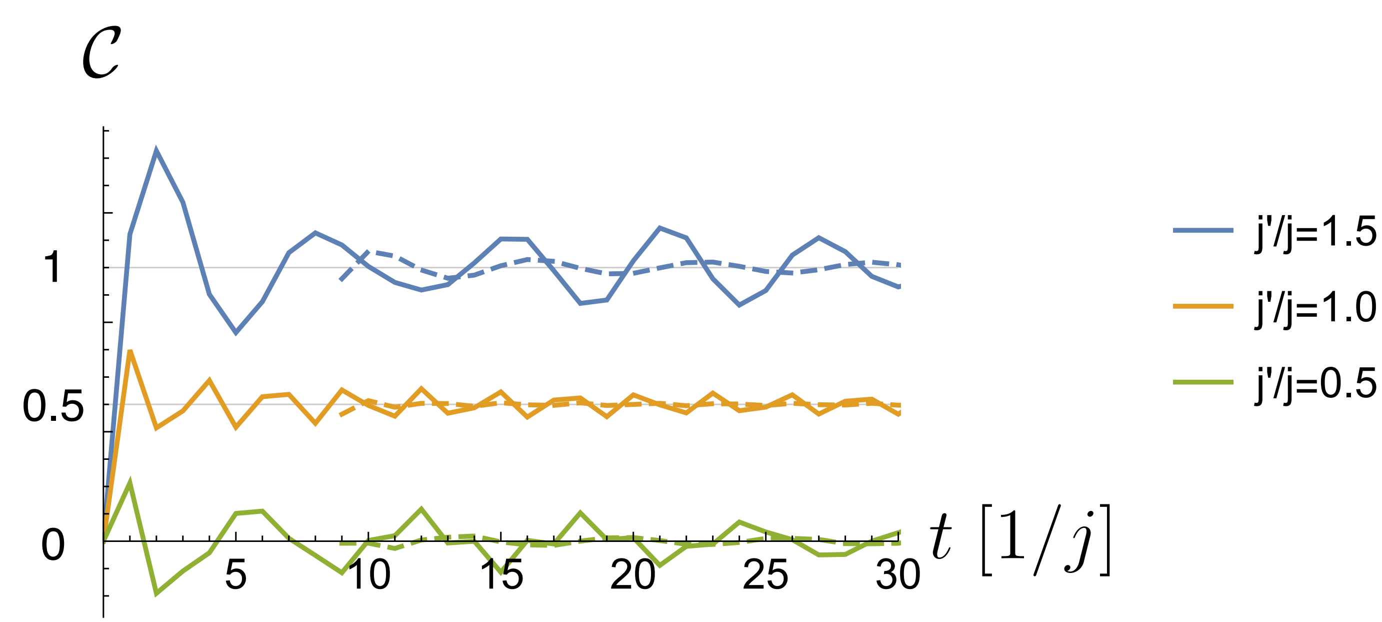
  localized on the $m=0$ cell:



- **Mean Chiral Displacement:** $\mathcal{C}(t) \equiv 2\langle\widehat{\Gamma m}(t)\rangle = 2\Big[\langle m_A(t)\rangle - \langle m_B(t)\rangle\Big]$

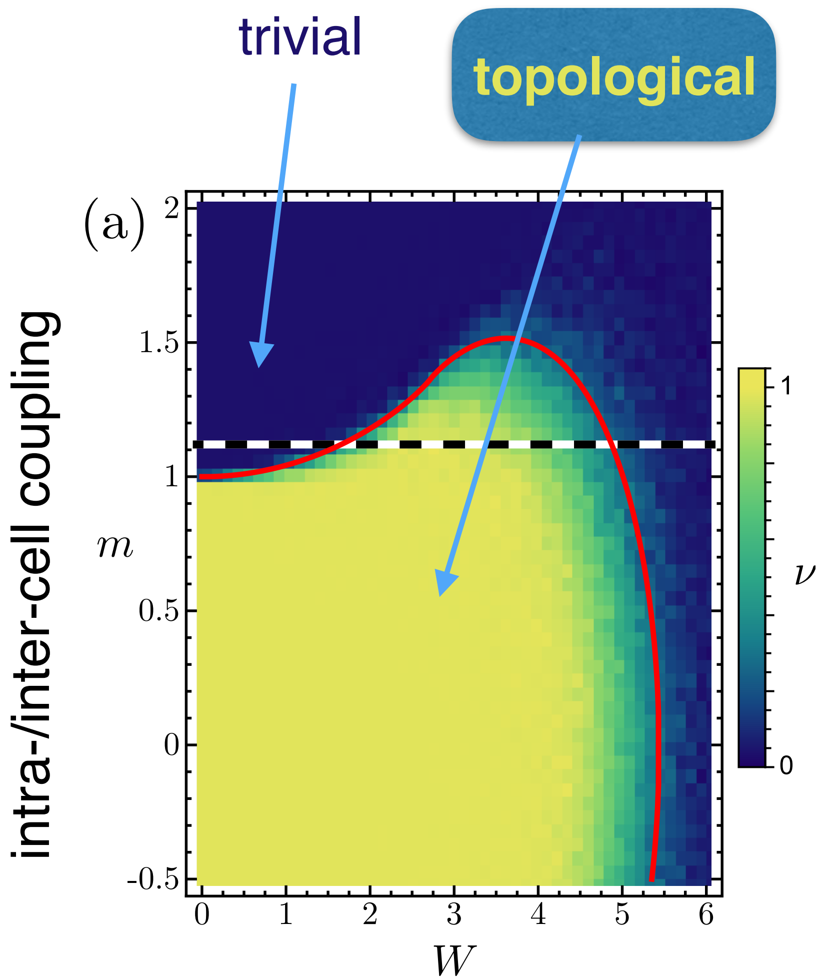
$$\mathcal{C}(t) = \oint \frac{\mathrm{d}k}{2\pi}\langle\psi(t)|2\sigma_z(i\partial_k)|\psi(t)\rangle = \mathcal{W} - \oint \frac{\mathrm{d}k}{2\pi}\cos(2\epsilon_k t)\partial_k\phi \xrightarrow{t\to\infty} \mathcal{W}$$

- *Bulk* measurement
- Fast convergence to $\mathcal{W}$
- Signals topological transitions!



Cardano, D'Errico, Dauphin, … Marrucci, Lewenstein & PM Nature Comm. (2017)

Adding disorder to a topological insulator

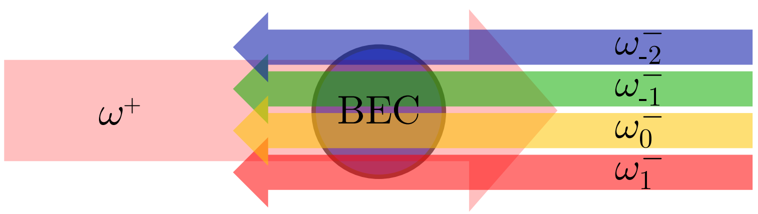


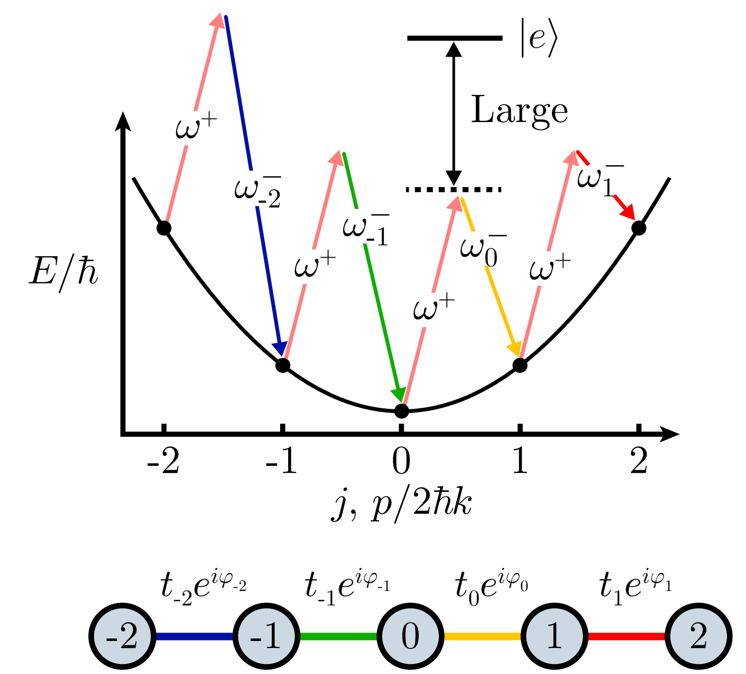
disorder strength

Meier, An, Dauphin, Maffei, PM, Taylor and Gadway, Science (2018)

Atomic wires

- Atomic, non-interacting BEC





- Laser-driven coupling of discrete-momentum states

- 1D Hubbard model with built-in chiral symmetry:

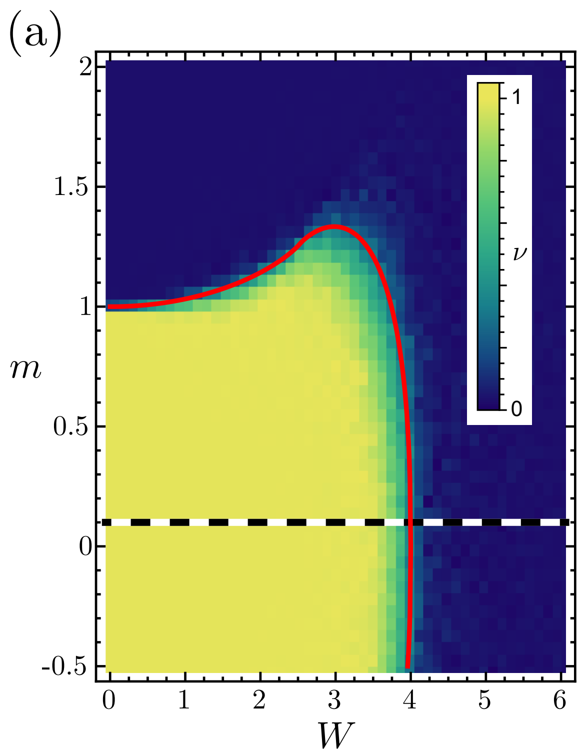
$$H_{\text{eff}} \approx \sum_j t_j (e^{i\varphi_j} |\widetilde{\psi}_{j+1}\rangle\langle\widetilde{\psi}_j| + \text{h.c.})$$

- Full control on each tunneling's strength and phase

Detecting topology

- A topological wire becomes trivial by adding disorder

intra-/inter-cell coupling



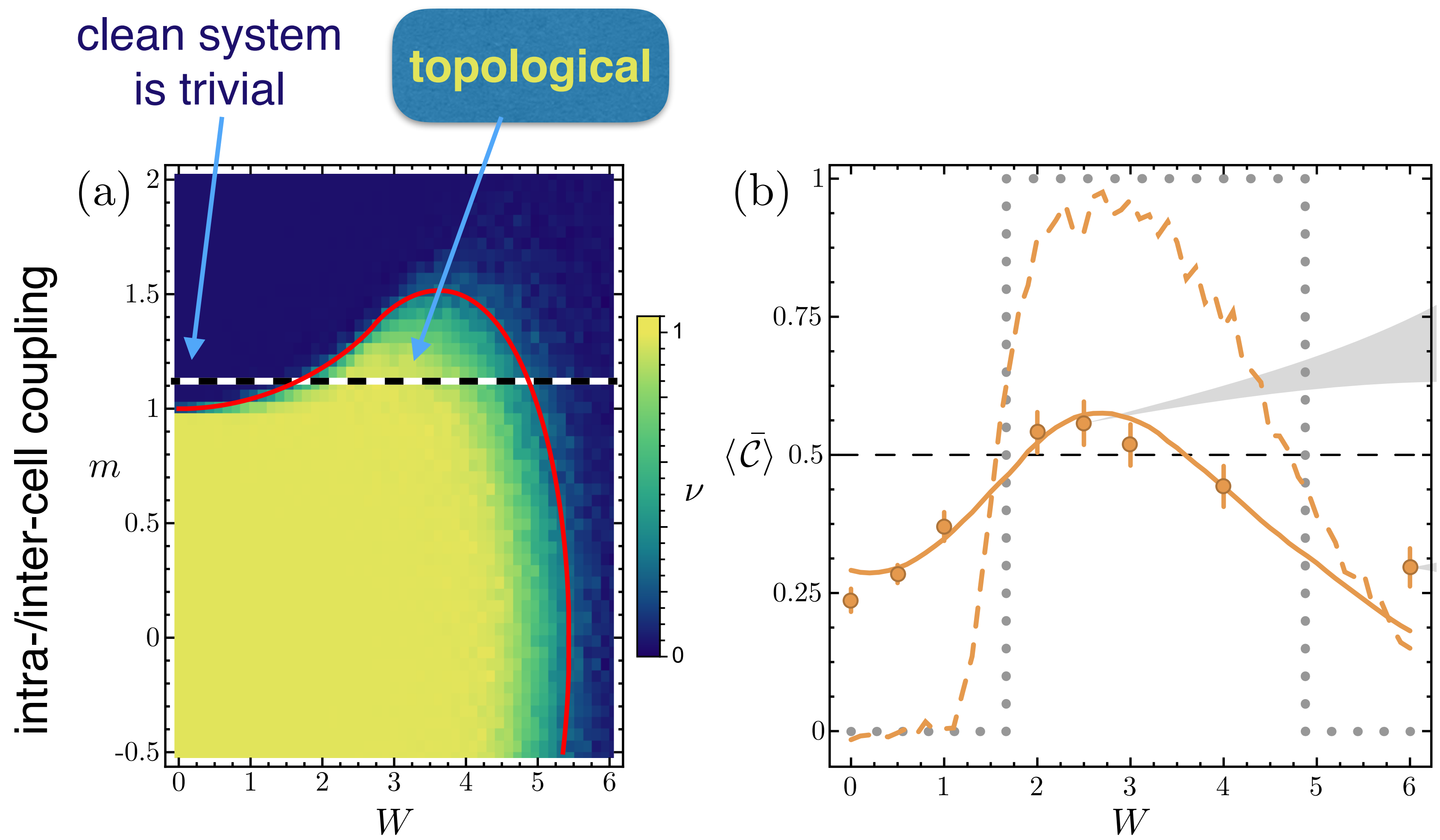
disorder strength

color map: real-space computation of the winding

red line: critical boundary (diverging localization length)

Topological Anderson transition

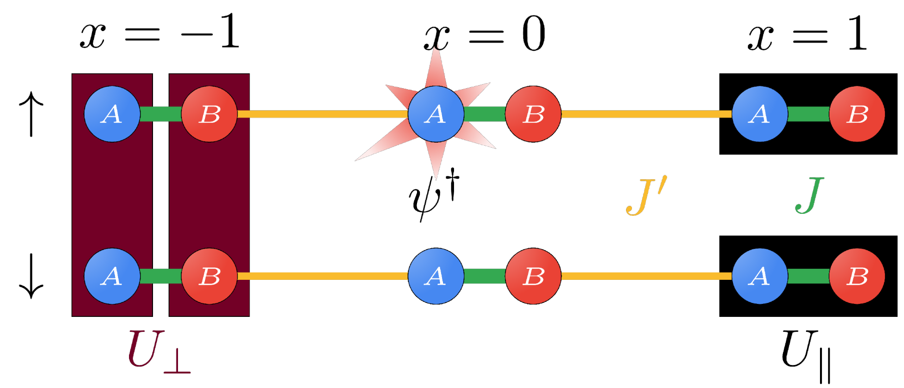
A trivial wire is driven into the topological phase by adding disorder



disorder strength

Meier, An, Dauphin, Maffei, PM, Taylor and Gadway,
Science (2018)

Interacting fermionic chains

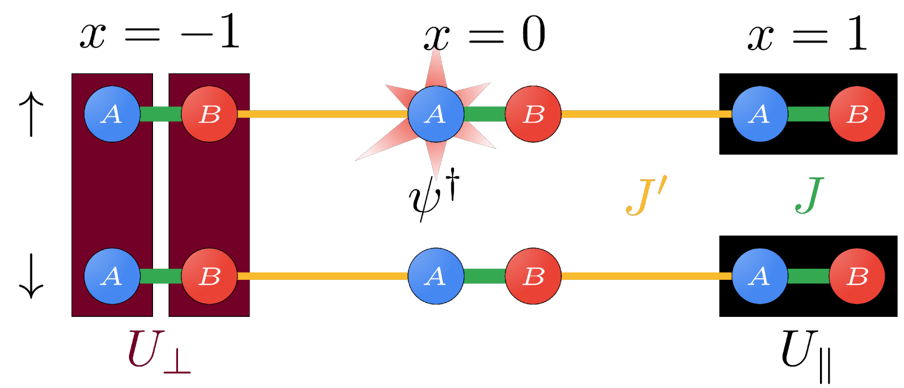


Half-filled Fermi sea
($U_\perp = U_\parallel = 0$)

The MCD equals the winding straight away (no oscillations!)

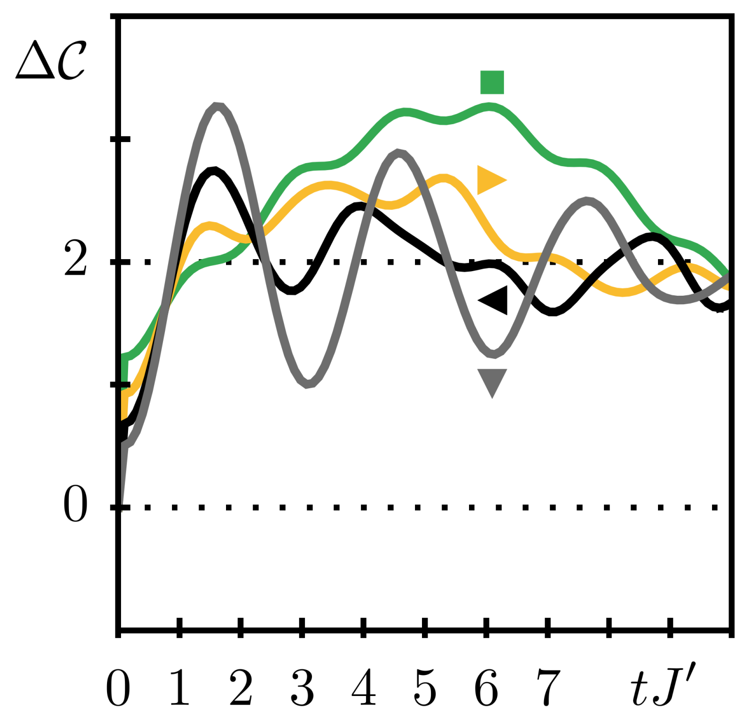
[Haller, PM and Rizzi, on the arXiv very soon]

Interacting fermionic chains

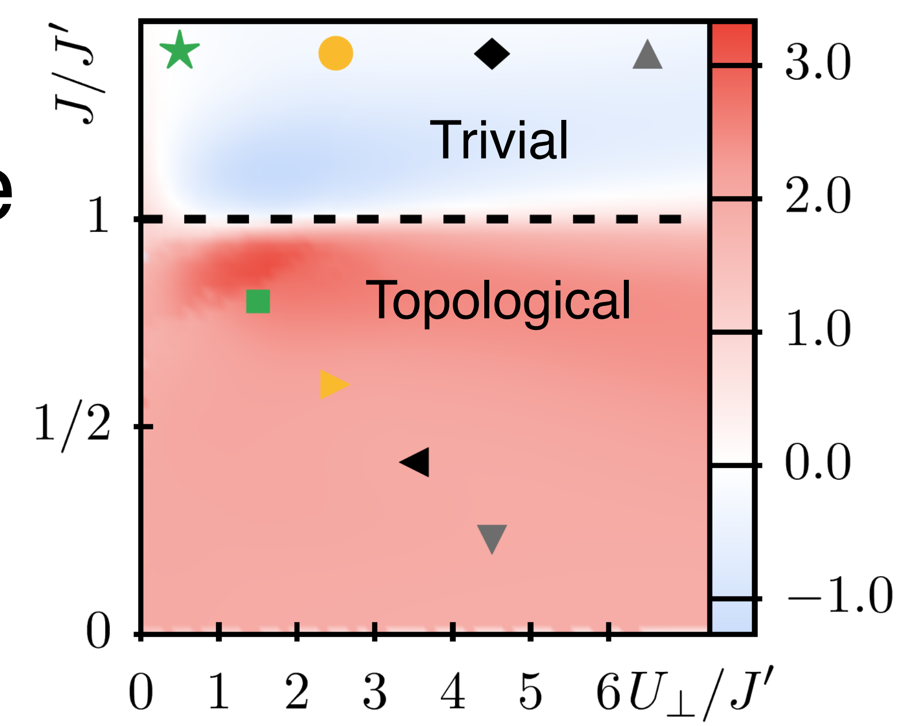


Peierls-Hubbard model:
$U_\perp$ only ($U_\parallel$=0)

MCD:
(in the topo. region)

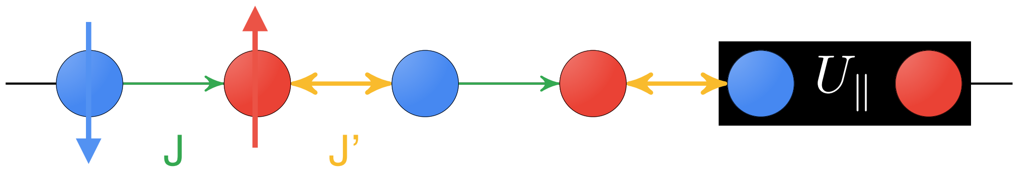


average
MCD:



[Haller, PM and Rizzi, on the arXiv very soon]

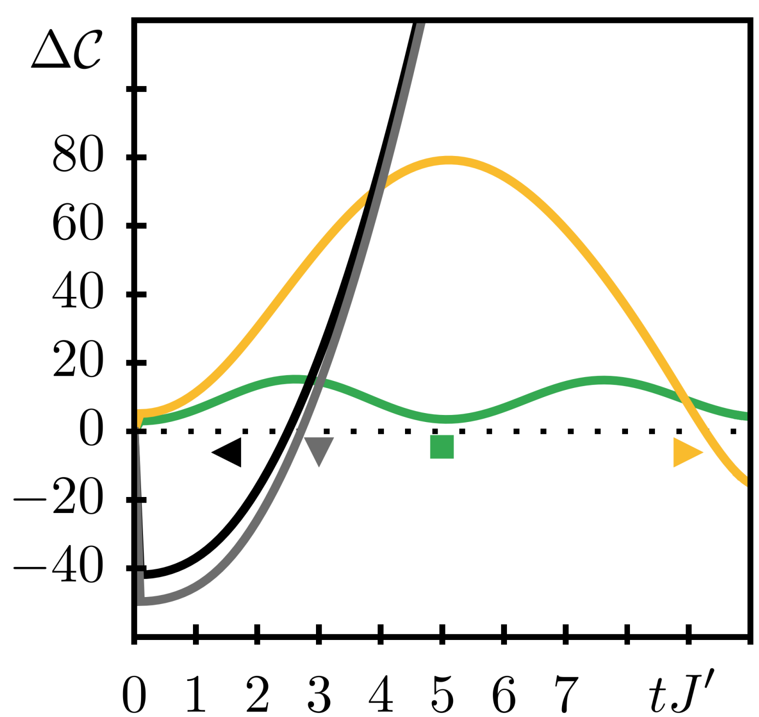
Interacting fermionic chains



$U_\parallel$ only ($U_\perp$=0)

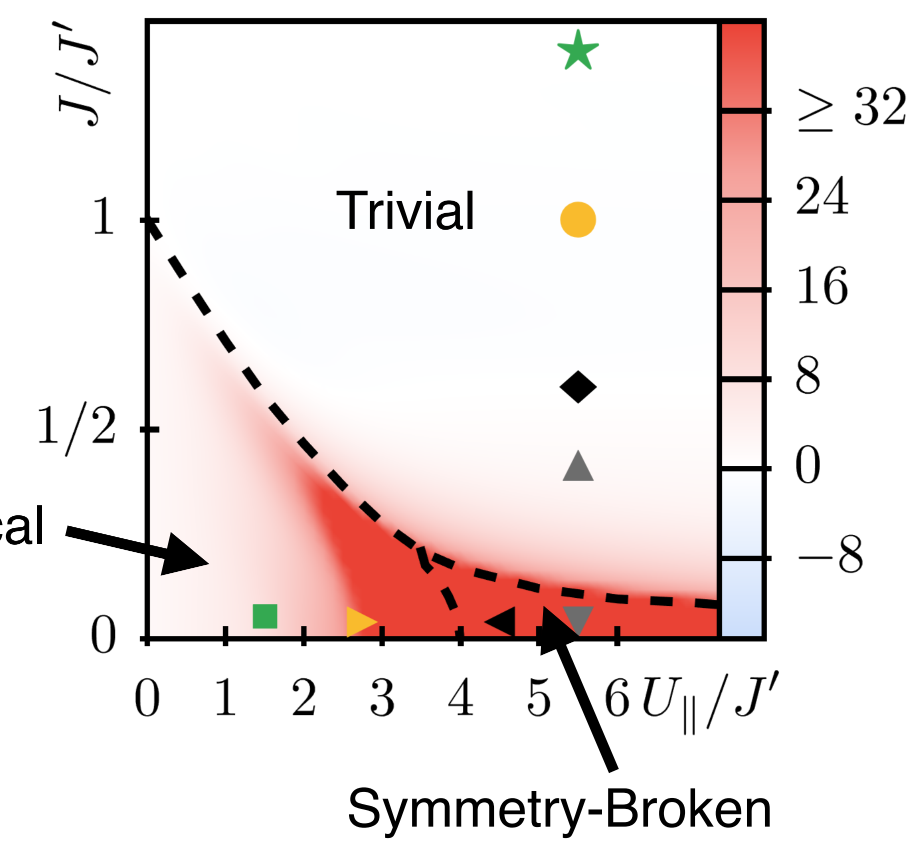
MCD:

(across the topo/SB transition)



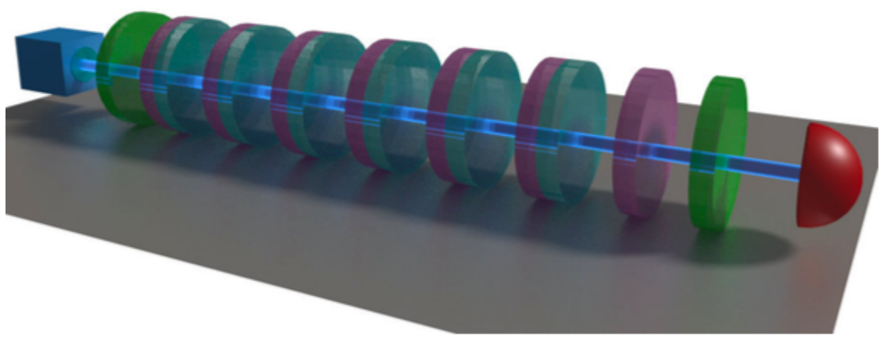
average *MCD:*

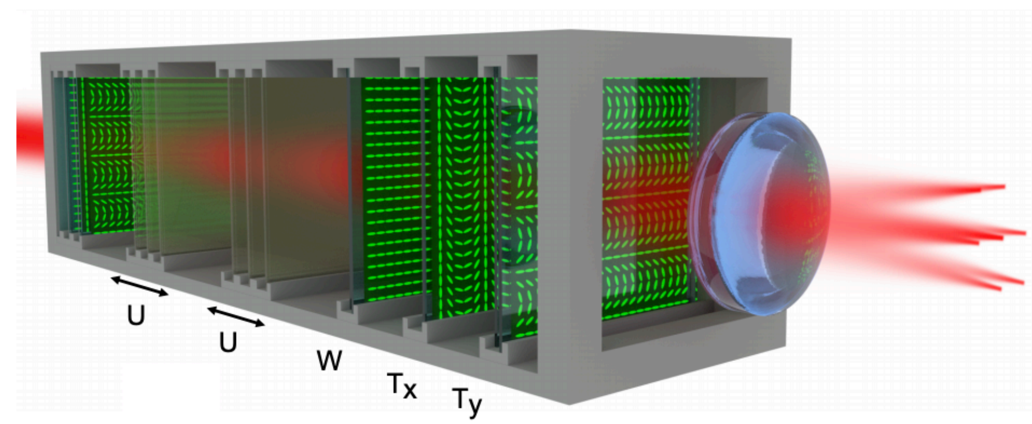
Topological



[Haller, PM and Rizzi, on the arXiv very soon]

Photonic quantum walks





1D

2D

Cardano, D'Errico, …, and PM, Nature Comm. 2017
D'Errico, Di Colandrea, PM, …, and Cardano, arXiv 2020

see Francesco's poster this afternoon,
and Alessio's talk on Thursday morning

quasi-periodicity of the energy spectrum
leads to two inequivalent invariants ⟷ edge states

D'Errico, Cardano, Esposito, …, PM *et al.*, Optica 2020

see earlier talk by Filippo

Collaborators

UPC

Theory


Maria Maffei


Alexandre Dauphin

JGU JOHANNES GUTENBERG UNIVERSITÄT MAINZ


Andreas Haller

ICFO Institut de Ciències Fotòniques


Maciej Lewenstein

Universität zu Köln


Matteo Rizzi

ULB UNIVERSITÉ LIBRE DE BRUXELLES

AARHUS UNIVERSITY


Nathan Goldman

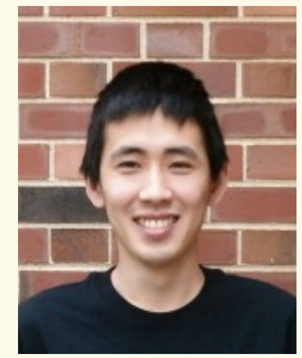

Arturo Camacho-Guardian


Georg Bruun

Experiments

atomic wires


Eric J. Meier

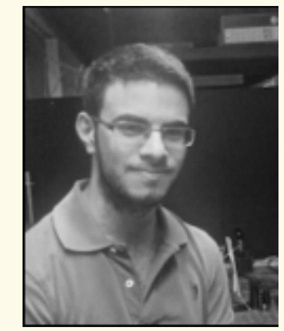

Fangzhao An

PHYSICS ILLINOIS


Bryce Gadway


Hughes Taylor

photonics


Alessio D'Errico


Filippo Cardano


Francesco di Colandrea


Chiara Esposito

SLAM group


Lorenzo Marrucci

Conclusions

- The ***mean chiral displacement*** is a **topological marker** capturing the winding of chiral systems
  (static, periodically driven, disordered, and interacting)

- Experimental observation of a **topological Anderson transition**

- Detect topology & symmetry-breaking in **interacting fermionic chains**

- Characterization of a **periodically-driven chiral model**
  (two independent invariants $\longleftrightarrow$ two kinds of edge states)

- Dynamical observables for *other topological classes*?

Floquet chiral models: Nature Comm. 2017
MCD theory: New J. Phys. 2018
Topo. Anderson Insulator: Science 2018
Topological polarons: Phys. Rev. B (Rapid Comm.) 2019
Quenched QWs: arXiv Jan 2020
Interacting chains: to be posted on arXiv very soon

Thank you!

Resistance to disorder

SSH model in the topological phase $\quad j' = 2j \quad \rightarrow \quad \begin{cases} \mathcal{W} = 1 \\ \Delta_{\text{gap}} = 2j \end{cases}$

+

independent disorder of amplitude Δ on ***all*** tunnelings

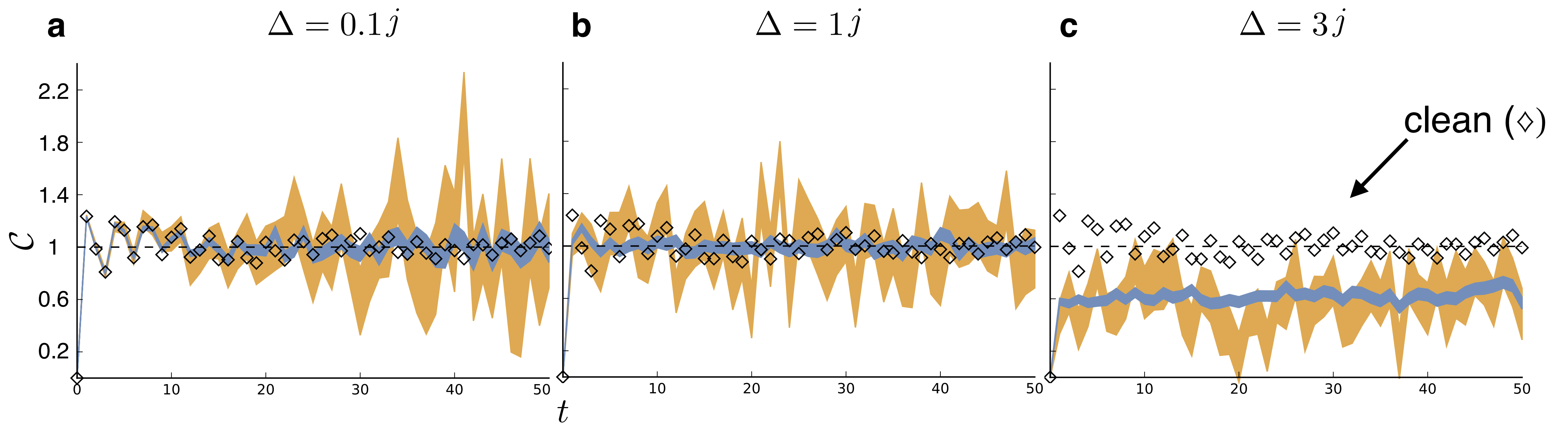
+

localized initial condition (randomly-polarized)

+

average over 50 (1000) disorder realizations

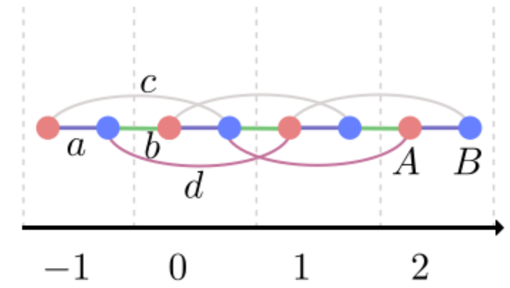
⬇

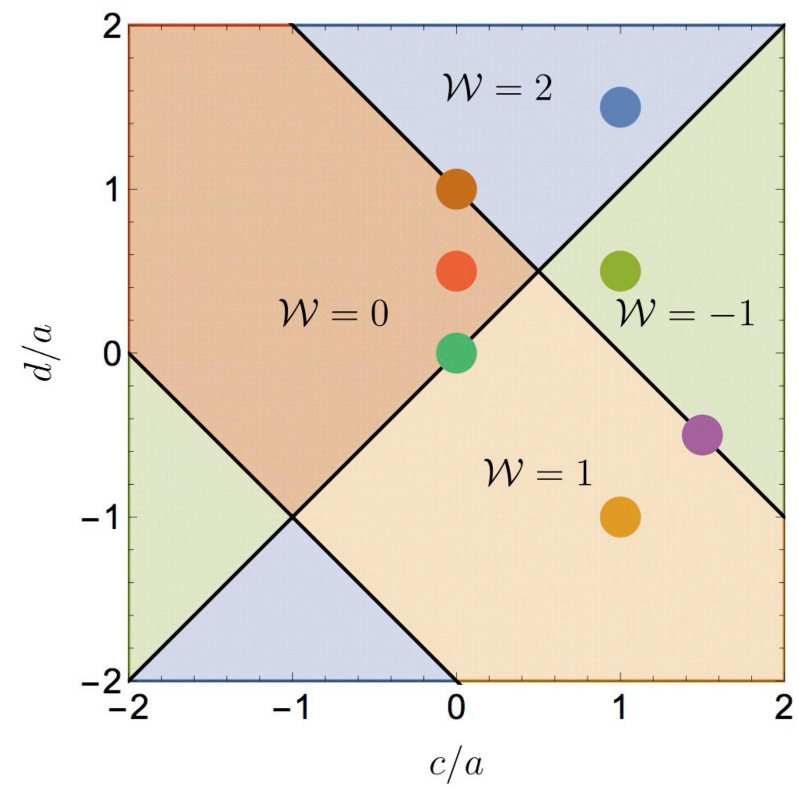


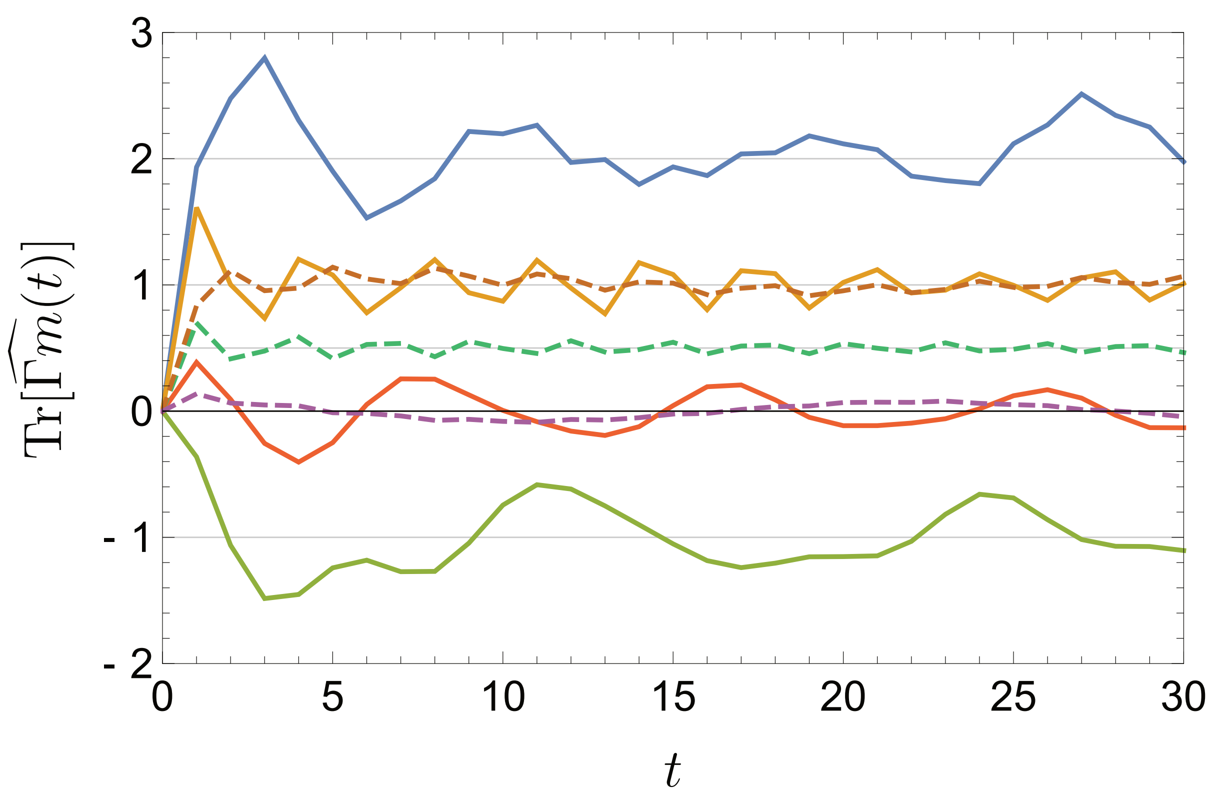
the MCD stays locked to the topological invariant as long as $\Delta < \Delta_{\text{gap}}$

Higher windings

- Extension to long-ranged models:



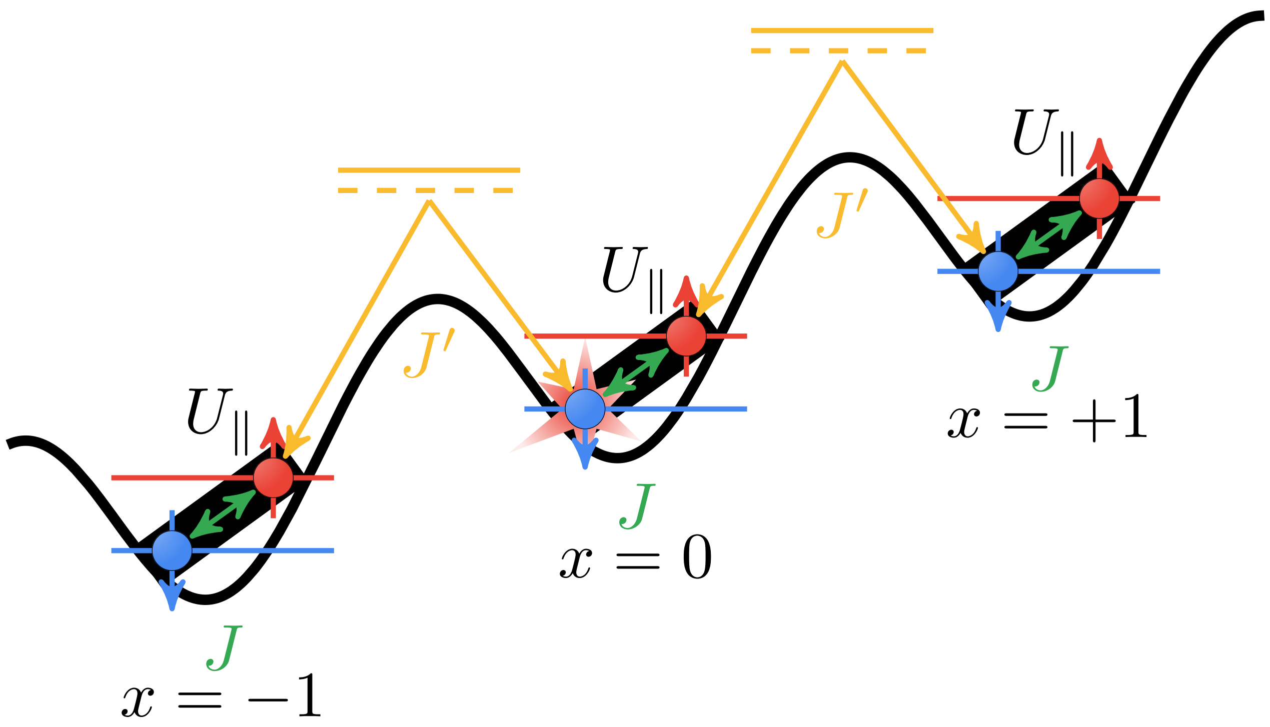




- At critical boundaries: MCD converges to the mean of the winding in the neighboring phases

Maffei, Dauphin, Cardano, Lewenstein & PM
New J. Phys. 2018

Experimental proposal



- tilted optical lattice blocks tunneling
- inter-cell tunneling J' restored by two-photon transitions
- intra-cell tunneling implemented by RF transitions